

Problem Set 9: NP-completeness

1. (EASY) Show that Vertex Cover \leq_p Independent Set
2. (EASY) In the HITTING SET problem, we are given a family of sets $\{S_1, S_2, \dots, S_n\}$ and a budget b , and we wish to find a set H of size $\leq b$ which intersects every S_i , if such an H exists. In other words, we want $H \cap S_i \neq \emptyset$ for all i . Show that HITTING SET is NP-complete. [Hint: You can assume that Set Cover is NP-Complete. Try to reduce Set Cover to Hitting Set]
3. (MEDIUM) In an undirected graph $G = (V, E)$, we say $D \subseteq V$ is a *dominating set* if every $v \in V$ is either in D or adjacent to at least one member of D . In the DOMINATING SET problem, the input is a graph and a budget b , and the aim is to find a dominating set in the graph of size at most b , if one exists.
Prove that this problem is NP-complete. [Hint: Assume that Vertex Cover is NP-Complete. Reduce Vertex Cover to Dominating Set]
4. (HARD) Let VC-DEC be the decision version of the vertex cover problem and VC-OPT be its optimisation version. Show that VC-DEC \leq_p VC-OPT and VC-OPT \leq_p VC-DEC