Assignment 1

Due Date: In class on August 23.

1. (10 points) **Dancing Partners**

   You are pairing couples for a very conservative formal ball. There are n men and m women, and you know the height and gender of each person there. Each dancing couple must be a man and a woman, and the man must be at least as tall as, but no more than 3 inches taller than, his partner. You wish to maximize the number of dancing couples given this constraint. Design an efficient algorithm that performs this task.

2. (10 points) **Multiple Classes**

   We are given a set of classes (with starting and finishing times) that have to be scheduled in a number of classrooms without conflicts. Design an algorithm to find the minimum number of classrooms needed for this purpose. Note that the timings are fixed and no two events can happen at the same time in the same hall. You can think about the events as intervals on the real line such that that we have to assign a colour to each interval in a way that no two overlapping intervals are assigned the same colour. What is the minimum number of colours required ?

3. (10 points) **Cookies**

   You are baby-sitting n children and have $m \geq n$ cookies to divide between them. You must give each child exactly one cookie (of course, you cannot give the same cookie to two different children). Each child has a greed factor $g_i, 1 \leq i \leq n$ which is the minimum size of a cookie that the child will be content with; and each cookie has a size $s_j, 1 \leq j \leq m$. Your goal is to maximize the number of content children, i.e., children i assigned a cookie j with $g_i \leq s_j$. Give a correct greedy algorithm for this problem, prove that it finds the optimal solution.

4. (20 points) **Verifying if an edge is in a MST**

   Suppose you are given an undirected graph G, with edge weights that you may assume are all distinct. G has n vertices and m edges. A particular edge e of G is specified. Give a algorithm with running time $O(m + n)$ to decide whether e is contained in a minimum-weight spanning tree of G.